Strengthened change point detection model for weak mean difference data

Shaoqian HUANG[1]¶, Qi ZHOU[2]¶ & Hongqing WANG[1]*

1 Department of Mathematics, Beijing Forestry University, Beijing, China

2 Department of Mathematics, Beijing Jiaotong University, Beijing, China

¶ These authors contributed equally to this work.

Conceptualization: Qi ZHOU

Data curation: Shaoqian HUANG.

Methodology: Qi ZHOU, Shaoqian HUANG,

Project administration: Hongqing WANG.

Software: Shaoqian HUANG

Supervision: Hongqing WANG.

Validation: Shaoqian HUANG, Qi ZHOU.

Writing – original draft: Qi ZHOU.

Writing – review & editing: Shaoqian HUANG, Qi ZHOU.

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Abstract

Objective: The lifetime difference in adjacent parallel structure components becomes small as the number of components belonging to the same parallel structure increases. To infer the system structure, we must clarify the components that belong to the same parallel structure.

Methods: A strengthened change point detection model (SCPDM) for weak mean difference data (WMDD) is established, which usually indicates that, as affected by a large variance, the mean difference in two subsignals for one data sequence becomes nonsignificant. For repeatedly retrievable WMDD, we performed two enhanced operations that doubled the mean difference by using the variance information and analyzed the asymptotic properties of the enhanced data. Then, we proposed an SCPDM based on the asymptotic results.

Results: Finally, we compared the SCPDM with two other main change point detection models and verified that the SCPDM is superior to other models using WMDD change point detection by the simulation method.

Limitations: This paper also have several limitations. First, we only discussed that $y_1,\ldots,y_T$ are independent with normal distribution and single change point. Second, the reason why the relationship between $|\mu_1 - \mu_2|$ and σ has an

important influence on the accuracy of change point detection is not discussed in depth. We only defined the ratio boundary of WMDD by experience and simulation.

Conclusions: Traditional change point detection models may become insensitive or ineffective for WMDD. We gave some asymptotic analysis and established a enhanced change point detection model (SCPDM) based on the asymptotic results. Compared with the traditional method, SCPDM can effectively detect the change point.

Keywords: single change point detection; weak mean difference; large variance; enhanced operations; simulation

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1. Introduction

Weak mean difference data (WMDD) are a type of data in which the information of the mean difference is reduced by a large variance. For example, as is shown in Fig 1, when the mean difference and the standard deviation , the location of the change point is easily detected. If the standard deviation , the accuracy of change point detection may be decreased. However, if the standard deviation , the location of the change point is hardly detected by most current models.

An important example of WMDD comes from the reliability field. Assume that the resistance values of all of the components in Fig 2. are equal. Jin et al. [46] pointed out that components which are tested in a laboratory environment differ in significant ways from those which have experienced operations in fielded systems where the homogeneous components will suffer different degrees of damage. As is shown below, the voltages of components belonging to adjacent parallel structures become more similar as the number of components belonging to the same parallel structure increases, which means that the lifetime difference in components belonging to adjacent parallel structures becomes small too. To infer the system structure, we must clarify the components belonging to the same parallel structure. As we all know, components in the same parallel structure have homogeneous life data. Hence, detecting the small lifetime difference in components is extremely important to distinguish whether components belong to the same parallel structure or not and is beneficial to establish a topology diagram of the system structure.

The difficulty of detecting change points for WMDD is how to capture the small differences between subsignals. To solve this problem, we performed two enhanced operations which increased the mean difference between subsignals by utilizing the information of the variance. In addition, we then analyzed the asymptotic property of the enhanced data. Next, we proposed a strengthened change point detection model (SCPDM) according to the asymptotic results. Finally, we compared the SCPDM with two current main models and verified that the SCPDM has a higher efficiency than those of the other models in the WMDD change point detection by the simulation method.

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2. Literature review

The so-called change point is the location at which a certain amount in the signal sample suddenly changes [1]. The change point often represents a qualitative change for our object of focus. Historically, Page [2-3] first proposed the study of change points in sample testing. To detect the change points in a signal sample, , the process roughly involves the following several steps. First, select an associated cost function [4] to measure the homogeneity in each subsignal. Second, according to whether the number of change points is fixed, compute a discrete optimal problem to obtain the estimations of the locations of the change points. In different change point detection models, establishing a suitable cost function for a specific sequence is the first and most important step [5]. Many classical change point detection models have been proposed for various kinds of signals, mainly including the following three types. For piecewise independent identically distributed (i.i.d.) signals, the mean shift model was first established for normal random signal samples which had a piecewise constant mean and constant variance [3, 6-9]. Second, certain signals may have their means shift along with shifts of their variances. For example, the mean shift and scale shift models were established for normal random signal samples with piecewise constant means and variances by certain predecessors [10-11]. Except for normal random samples, the rate shift model has been studied for Poisson distribution signals with piecewise constant rate parameters [12-13]. The second type of change point model is appropriate for a signal with a linear dependency between the variables along with changes happening at certain unknown instances, which are also called structural changes [14-16]. In this situation, several well-known models were established, such as the autoregressive model [1, 17] and multiple regression models [16, 18]. Other commonly used change point detection models include kernel change point detection [19-22] and the Mahalanobis-type metric [23]. Kernel change point detection can be operated on the high-dimensional mapping of the original signal that is implicitly defined by a kernel function. Certain machine learning techniques may be involved in this kind of method, such as a support vector machine or clustering [24-25]. In addition, in certain clustering methods, the Mahalanobis-type metric is usually used to replace the cost function in the mean shift model [26-28].

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Except for the models mentioned above, several classical models based on algorithms have been proposed about inferring change points, mainly including the following four types [10]. The first model is based on the likelihood ratio. Csörgö and Horváth [29] established a change point detection model under the assumption of a multivariate Gaussian distribution. This model is mainly used in analyzing the change points in time series data. The second model is based on the Bayes model. A number of researchers have focused on this method. Kander and Zacks [30] aimed at the exponential family to establish a change point detection model, while Gardner [31] established a model based on the normal distribution. Later, the model was extended to the large sample distribution theory, multivariate normal distribution and general linear regression field [9, 32-33]. The third model is based on the maximum likelihood. This kind of method has the mature large sample theory. For example, Fotopoulos et al. [34-35] established the exact computable expressions, bounds and approximations for certain analysis results. The last model is based on samples. This kind of method focused on the nonparametric method which has the advantage of being distribution free.

Among the subdirections of change point detection, signal samples with mean shifts have always been a research hot spot. Hawkins et al. [36] used the sample variance without degrees of freedom to determine the change point. In [37], the maximum likelihood estimation method was utilized to analyze the change point under the premise of verifying the type of population distribution. Later, prior knowledge was incorporated in establishing the change point detection model in [38]. In [39], the change point location was determined by analyzing the local information near a point, which involved complex distribution information usually substituted for by certain approximate results. In recent years, certain new methods also have been discussed regarding change point detection. As indicated in [40], an optimal algorithm was introduced to determine the location of a change point. In [41], an adapted algorithm was established by the polynomial maximization method. In [42], partition models were set up for testing the existence of a mean shift and estimating the location of the change point. In addition, lasso methods were established and improved by many authors [43-45].

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3. Methodology (Design/Approach)

3.1 Two enhanced operations

Because the standard deviation is far larger than the mean difference, it is unwise and inefficient to perform change point detection on WMDD using traditional models. In fact, compared with the information of the mean difference, the variance is very remarkable which may supply more information. Therefore, we considered utilizing the variance that belongs to a kind of disturbance information in analyzing change points by performing the following two enhancements.

Now, we believe that t^* ($t^* \in \{2,...,T\}$) is the only abrupt location in sequence $y_1,...,y_T$. For $y_1,...,y_T$, we first conducted the first operation called enhanced-Ⅰ at t ($t \in \{2,...,T\}$) and obtained the enhanced-Ⅰ sequence $y_1^{'},...,y_T^{'}$.

$$y_1^{'} = y_1,\ y_i^{'} = \min(y_{i-1}, y_i),\ i = 1,...,t-1,$$

$$y_t^{'} = y_t,\ y_i^{'} = \max(y_{i-1}, y_i),\ i = t+1,...,T.$$

Then, we conducted the second operation called enhanced-Ⅱ at t ($t \in \{2,...,T\}$) and obtained the enhanced-Ⅱ sequence $y_1^{''},...,y_T^{''}$.

$$y_1^{''} = y_1,\ y_i^{''} = \max(y_{i-1}, y_i),\ i = 1,...,t-1,$$

$$y_t^{''} = y_t,\ y_i^{''} = \min(y_{i-1}, y_i),\ i = t+1,...,T.$$

Intuitively, these kinds of enhancements utilize the variance information directly through taking large or one smaller between adjacent samples.

Now, we provide several symbolic explanations. For a signal sample $y_1,...,y_T$, we performed the above two operations at location t ($t \in \{2,...,T\}$), where $\mu_1^{'}(t, y^{'})$ indicates the sample mean of $y_1^{'},...,y_{t-1}^{'}$; $\mu_2^{'}(t, y^{'})$ indicates the sample mean of $y_t^{'},...,y_T^{'}$; $\mu_1^{''}(t, y^{''})$ indicates the sample mean of

$y_1^{"},...,y_{t-1}^{"}$; and $\mu_2^{"}(t,y^{"})$ indicates the sample mean of $y_t^{"},...,y_T^{"}$; $\mu_1^{"}(t,y^{"})$ indicates the sample mean of $y_1^{"},...,y_{t-1}^{"}$. In addition, $\cup$ and $\bullet$ represent operations on the homogeneous signal sample and the signal sample containing a change point, respectively.

3.2 Asymptotic property of the enhanced sequence

We considered establishing a certain asymptotic property of the enhanced sequence in the following theorem. First, we will explain several symbols. Assume that Y_t, with ($t=1,...,T$), are independent random variables, with probability distribution $f(y_t \mid \theta)$, and θ is a vector-valued parameter. $y_1,...,y_T$ is a set of samples of $Y_1,...,Y_T$. For a signal sample that has one change location at t^* ($t^* \in \{2,...,T\}$), μ_1 indicates the population mean of the first subsignal and μ_2 indicates the population mean of the second subsignal. σ^2 indicates the constant population variance. Generally, WMDD indicate that $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$.

Theorem 1. Assume that $y_1,...,y_T$ is i.i.d., i.e., homogeneous with a constant variance σ^2. The above two operations are performed n_c times independently at location t ($t \in \{2,...,T\}$). Then, we have the following asymptotic property:

$$P\left(\lim_{n_c,t,T-t\to\infty} \frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left|\mathring{\mu}_{1i}^{'}(t,y^{'}) - \mathring{\mu}_{2i}^{'}(t,y^{'})\right| - \left|\mathring{\mu}_{1i}^{"}(t,y^{"}) - \mathring{\mu}_{2i}^{"}(t,y^{"})\right|\right) = 0\right) = 1.$$

Proof. $y_1,...,y_T$ is i.i.d. According to the law of large numbers, we have the following:

$$\mathring{\mu}_{1i}^{'}(t,y^{'}) \to \mu_{\min} \quad a.s. \quad t \to \infty,$$

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where $\mu_{\min}$ is the smaller population mean of the enhanced sequence.

Likewise, we have the following:

$$\overset{\circ}{\mu}_{2i}{}'(t,y') \to \mu_{\max} \quad a.s. \quad T-t \to \infty$$

where $\mu_{\max}$ is the larger population mean of the enhanced sequence. As a result, we have the following:

$$\overset{\circ}{\mu}_{1i}{}'(t,y') - \overset{\circ}{\mu}_{2i}{}'(t,y') \to \mu_{\min} - \mu_{\max} \quad a.s. \quad t, T-t \to \infty$$

On the other hand:

$$\overset{\circ}{\mu}_{1i}{}''(t,y'') \to \mu_{\max} \quad a.s. \quad t \to \infty$$

$$\overset{\circ}{\mu}_{2i}{}''(t,y'') \to \mu_{\min} \quad a.s. \quad T\text{-}t \to \infty$$

Consequently:

$$\overset{\circ}{\mu}_{1i}{}''(t,y'') - \overset{\circ}{\mu}_{2i}{}''(t,y'') \to \mu_{\max} - \mu_{\min} \quad a.s. \quad t, T\text{-}t \to \infty$$

In addition, $f(x,y)=|x|-|y|$ is a continuous function, so the $a.s.$ converge can be preserved under the transformation of this function, so we have the following:

$$\left|\overset{\circ}{\mu}_{1i}{}'(t,y') - \overset{\circ}{\mu}_{2i}{}'(t,y')\right| - \left|\overset{\circ}{\mu}_{1i}{}''(t,y'') - \overset{\circ}{\mu}_{2i}{}''(t,y'')\right| \to |\mu_{\min} - \mu_{\max}| - |\mu_{\max} - \mu_{\min}| = 0 \quad a.s. \quad t, T\text{-}t \to \infty$$

Thus, because the process is independently performed n_c times, $\left|\overset{\circ}{\mu}_{1i}{}'(t,y') - \overset{\circ}{\mu}_{2i}{}'(t,y')\right| - \left|\overset{\circ}{\mu}_{1i}{}''(t,y'') - \overset{\circ}{\mu}_{2i}{}''(t,y'')\right|$ ($i = 1,\ldots,n_c$) can be viewed as independently and identically distributed. According to the law of large number, we have the following:

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left|\overset{\circ}{\mu}_{1i}{}'(t,y') - \overset{\circ}{\mu}_{1i}{}'(t,y')\right| - \left|\overset{\circ}{\mu}_{1i}{}''(t,y'') - \overset{\circ}{\mu}_{1i}{}''(t,y'')\right|\right) \to 0 \quad a.s. \quad t, T\text{-}t \to \infty, n_c \to \infty$$

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Remark 1. Theorem 1 demonstrates that when $y_1,...,y_T$ is homogeneous and the above two operations are performed n_c times independently at any location t ($t \in \{2,...,T\}$), the value of $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left|\overset{\circ}{\mu}_{1i}''(t,y')-\overset{\circ}{\mu}_{2i}''(t,y')\right|-\left|\overset{\circ}{\mu}_{1i}''(t,y'')-\overset{\circ}{\mu}_{2i}''(t,y'')\right|\right)$ fluctuates at approximately 0.

Next, we will give the asymptotic property when there is only one change point among the signal samples. We will present the asymptotic results at the position of the change point.

Theorem 2. Assume that the independent $y_1,...,y_T$ only has one change point t^* with a constant variance σ^2. At t^*, the above two enhanced operations are independently performed n_c times. Then, we have the following asymptotic property:

(1) If $\frac{|\mu_1-\mu_2|}{\sigma}>1$, we then have:

$$P\left(\lim_{n_c,t^*,T-t^*\to\infty}\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{2i}''(t^*,y'')\right\|-\left(\left|\overset{\circ}{\mu}_{1i}'(t^*,y')-\overset{\circ}{\mu}_{2i}'(t^*,y')\right|+\left|\overset{\circ}{\mu}_{1i}''(t^*,y'')-\overset{\circ}{\mu}_{2i}''(t^*,y'')\right|\right)\right)=0\right)=1,$$

The meanings of $\overset{\circ}{\mu}_{1i}'(t^*,y')$, $\overset{\circ}{\mu}_{2i}'(t^*,y')$, $\overset{\circ}{\mu}_{1i}''(t^*,y'')$ and $\overset{\circ}{\mu}_{2i}''(t^*,y'')$ are the same as in **Theorem 2**.

(2) If $\frac{|\mu_1-\mu_2|}{\sigma}\le 1$, we then have:

$$P\left(\lim_{n_c,t^*,T-t^*\to\infty}\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{1i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{1i}''(t^*,y'')\right\|\right)=2|\mu_2-\mu_1|\right)=1$$

Proof. (1) First, we prove the first part. The following formula is clear when $\mu_1<\mu_2$

$$\left|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\overset{\circ}{\mu}_{1i}'(t^*,y')-\overset{\circ}{\mu}_{2i}'(t^*,y')\right|\to(\mu_2-\mu_1)\quad a.s.\quad t^*,T-t^*\to\infty$$

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Because $\frac{|\mu_1 - \mu_2|}{\sigma} > 1$, an incorrect enhanced operation cannot reverse the direction of the mean difference, so we have the following:

$$\left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{2i}''(t^*, y'')\right| + \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right| \to (\mu_2 - \mu_1) \quad a.s. \quad t^*, T - t^* \to \infty$$

Likewise, when $\mu_1 > \mu_2$, we have the following two relationships:

$$\left|\dot{\mu}_{1i}'(t^*, y') - \dot{\mu}_{2i}'(t^*, y')\right| + \left|\mathring{\mu}_{1i}'(t^*, y') - \mathring{\mu}_{2i}'(t^*, y')\right| \to (\mu_1 - \mu_2) \quad a.s. \quad t^*, T - t^* \to \infty$$

$$\left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{2i}''(t^*, y'')\right| - \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right| \to (\mu_1 - \mu_2) \quad a.s. \quad t^*, T - t^* \to \infty$$

In summary, we have the following:

$$\left\|\dot{\mu}_{1i}'(t^*, y') - \dot{\mu}_{2i}'(t^*, y')\right| - \left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{2i}''(t^*, y'')\right\| - \left(\left|\mathring{\mu}_{1i}'(t^*, y') - \mathring{\mu}_{2i}'(t^*, y')\right| + \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right|\right) \to 0 \quad a.s. \quad t^*, T - t^* \to \infty$$

Thus, because the process is independently performed n_c times, $\left\|\dot{\mu}_{1i}'(t^*, y') - \dot{\mu}_{2i}'(t^*, y')\right| - \left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{2i}''(t^*, y'')\right\| - \left(\left|\mathring{\mu}_{1i}'(t^*, y') - \mathring{\mu}_{2i}'(t^*, y')\right| + \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right|\right)$ $(i = 1, \ldots, n_c)$ can be viewed as being independent and identically distributed. According to the law of large numbers, we have the following:

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*, y') - \dot{\mu}_{2i}'(t^*, y')\right| - \left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{1i}''(t^*, y'')\right\| - \left(\left|\mathring{\mu}_{1i}'(t^*, y') - \mathring{\mu}_{2i}'(t^*, y')\right| + \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right|\right)\right) \to 0 \quad a.s. \quad t^*, T - t^*, n_c \to \infty$$

Proof. (2) The following formula is clear when $\mu_1 < \mu_2$,

$$\left|\dot{\mu}_{1i}'(t, y') - \dot{\mu}_{2i}'(t, y')\right| - \left|\mathring{\mu}_{1i}'(t, y') - \mathring{\mu}_{2i}'(t, y')\right| \to (\mu_2 - \mu_1) \quad a.s. \quad t^*, T - t^* \to \infty$$

Due to $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$, the enhanced-Ⅱ operation can reverse the mean difference, so we have the following:

$$\left|\dot{\mu}_{1i}''(t^*, y'') - \dot{\mu}_{2i}''(t^*, y'')\right| - \left|\mathring{\mu}_{1i}''(t^*, y'') - \mathring{\mu}_{2i}''(t^*, y'')\right| \to -(\mu_2 - \mu_1) \quad a.s. \quad t^*, T - t^* \to \infty$$

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Likewise, when $\mu_1 > \mu_2$, we have the following two relationships:

$$\left|\dot{\mu}_{1i}'(t^*,y') - \dot{\mu}_{2i}'(t^*,y')\right| - \left|\mathring{\mu}_{1i}'(t^*,y') - \mathring{\mu}_{2i}'(t^*,y')\right| \to -(\mu_1 - \mu_2) \quad a.s. \quad t^*, T-t^* \to \infty$$

$$\left|\dot{\mu}_{1i}''(t^*,y'') - \dot{\mu}_{2i}''(t^*,y'')\right| - \left|\mathring{\mu}_{1i}''(t^*,y'') - \mathring{\mu}_{2i}''(t^*,y'')\right| \to (\mu_1 - \mu_2) \quad a.s. \quad t^*, T-t^* \to \infty$$

Thus,

$$\left|\dot{\mu}_{1i}'(t^*,y') - \dot{\mu}_{2i}'(t^*,y')\right| - \left|\dot{\mu}_{1i}''(t^*,y'') - \dot{\mu}_{2i}''(t^*,y'')\right| \to 2|\mu_2 - \mu_1| \quad a.s. \quad t^*, T-t^* \to \infty$$

Because the process is independently performed n_c times, $\left\|\dot{\mu}_{1i}'(t^*,y') - \dot{\mu}_{2i}'(t^*,y')\right| - \left|\dot{\mu}_{1i}''(t^*,y'') - \dot{\mu}_{2i}''(t^*,y'')\right\|$ ($i = 1,\ldots,n_c$) can be viewed as being independently and identically distributed. Therefore, we have the following:

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y') - \dot{\mu}_{2i}'(t^*,y')\right| - \left|\dot{\mu}_{1i}''(t^*,y'') - \dot{\mu}_{2i}''(t^*,y'')\right\|\right) \to 2|\mu_2 - \mu_1| \quad a.s. \quad n_c, t, T\text{-}t \to \infty$$

Remark. 2. For WMDD, **theorem 2** demonstrates that when $y_1,\ldots,y_T$ has a change at t^* ($t^* \in \{2,\ldots,T\}$) and if the above two operations are performed n_c times independently at t^* ($t^* \in \{2,\ldots,T\}$), the value of $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y') - \dot{\mu}_{2i}'(t^*,y')\right| - \left|\dot{\mu}_{1i}''(t^*,y'') - \dot{\mu}_{1i}''(t^*,y'')\right\|\right)$ will reach the maximum value $2|\mu_2 - \mu_1|$; if the above two operations are performed n_c times independently at any nonchange location, the value of

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$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{1i}''(t^*,y'')\right\|\right)$ will be less than the maximum value $2|\mu_2-\mu_1|$. The latter finding is verified in Section 3 in detail.

3.3 SCPDM

For WMDD, the different asymptotic properties in **theorem 1** and **theorem 2** are important information for judging whether there is a change point and the location of the point. Consequently, we proposed a model to detect the change point in WMDD by sampling repeatedly in this section.

Assume that there is only one change point whose location is t^* $(1<t^*\le T)$ in $y_1,...,y_T$. To obtain a better estimate of t^*, we should establish a contrast function [5] to measure the goodness-of-fit of the signal. First, at the location of t ($t=2,...,T$), the above two enhanced operations are performed n_c times. The following contrast function is established:

$$\hat{t}^*(y)=\arg\max\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t,y')-\dot{\mu}_{2i}'(t,y')\right|-\left|\dot{\mu}_{1i}''(t,y'')-\dot{\mu}_{1i}''(t,y'')\right\|\right)$$

According to **theorem 2** and **theorem 3**, the value of $\left\|\dot{\mu}_{1t}'-\dot{\mu}_{2t}'\right|-\left|\dot{\mu}_{1t}''-\dot{\mu}_{2t}''\right\|$ is expected to be large when $\hat{t}^*=t^*$ and small when t^* is not well-estimated.

Throughout the establishment process of the SCPDM, how to solve this discrete optimization problem becomes clear. We only need to calculate $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t,y')-\dot{\mu}_{2i}'(t,y')\right|-\left|\dot{\mu}_{1i}''(t,y'')-\dot{\mu}_{1i}''(t,y'')\right\|\right)$ at ($t=2,...,T$), and make

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$\hat{t}^*(y) = \arg\max \frac{1}{n_c} \sum_{i=1}^{n_c} \left(\left\| \dot{\mu}_{1i}^{'}(t, y^{'}) - \dot{\mu}_{2i}^{'}(t, y^{'}) \right| - \left| \dot{\mu}_{1i}^{''}(t, y^{''}) - \dot{\mu}_{1i}^{''}(t, y^{''}) \right\| \right)$. Details can be seen in the next section. In the next section, we performed several simulation studies to estimate t^* under a normal distribution with various kinds of parameters.

4. Results

In this section, we will perform simulations in two parts. In the first part, we will verify the correctness of **theorem 2** and **remark 2**. In the second part, we performed several simulation studies to estimate t^* under a normal distribution with different parameters and compared the SCPDM with two current main models, verifying that the SCPDM has a higher efficiency than those of the other models for WMDD.

4.1 Verifying the correctness of the theorem

To verify the correctness of theorem 2, we generated random numbers based on the normal distribution, and the parameter settings are shown in the caption of each figure. For the public parameters, we set $T = 1000$, $t^* = 501$, and $n_c = 1000$.

For $\frac{|\mu_1 - \mu_2|}{\sigma} > 1$, as is shown in Fig 3, we carried out two sets of simulation analyses.

To verify the correctness of **remark 2**, when $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$, at the location of t ($t = 400,...,600$), $\frac{1}{n_c} \sum_{i=1}^{n_c} \left(\left\| \dot{\mu}_{1i}^{'}(t^*, y^{'}) - \dot{\mu}_{2i}^{'}(t^*, y^{'}) \right| - \left| \dot{\mu}_{1i}^{''}(t^*, y^{''}) - \dot{\mu}_{1i}^{''}(t^*, y^{''}) \right\| \right)$ is calculated and the results are shown in Fig 5.

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It can be seen from Fig. 5 that when t is near $t^* = 501$, $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}^{'}(t,y^{'}) - \dot{\mu}_{2i}^{'}(t,y^{'})\right| - \left|\dot{\mu}_{1i}^{''}(t,y^{''}) - \dot{\mu}_{1i}^{''}(t,y^{''})\right\|\right)$ reaches a maximum value, which is approximately $2|\mu_2 - \mu_1|$. The simulation result is consistent with **theorem 2**.

5. Discussion

When $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$, in order to verify that the SCPDM's estimation of t^* is better than those of the other models, we present certain simulated results based on traditional models, including the least squares model [47] and Bayes method [48]. We generated random samples $y_1,...,y_T$ based on the normal distribution, and the parameter settings are shown in the corresponding figure; we set the public parameters, namely, $T = 1000$, $t^* = 500$, and $n_c = 1000$. For the three models with the same parameter settings, we repeated the same operation 1000 times, counted the estimation results of t^* and regarded the frequency of each $\hat{t}^*$ as the probability of becoming a real change point, which reflected the accuracy of each model. The results are shown in Fig 6, Fig 7 and Fig 8.

By setting different parameters, the detection accuracies of the change point interval, i.e., $\Pr(\hat{t}^* \in [499,503])$, and change point location, i.e., $\Pr(\hat{t}^* = 501)$, of the three models were compared in Table 1 and Table 2, respectively.

When $\frac{|\mu_1 - \mu_2|}{\sigma} > 1$, i.e., the parameters were set to $\sigma = 0.5$ and $|\mu_1 - \mu_2| = 1$ for 1000 repeated tests, the Bayes model resulted in $\Pr(\hat{t}^* \in [499,503]) = 0.87$ and $\Pr(\hat{t}^* = 501) = 0.61$,

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and the least squares model resulted in $\Pr(\hat{t}^* \in [499,503]) = 0.942$ and $\Pr(\hat{t}^* = 501) = 0.561$. Both methods had a high accuracy for change point detection.

When $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$, i.e., the type of data is WMDD and we consider $\Pr(\hat{t}^* \in [499,503])$.

When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 1$, the accuracy of the SCPDM is 31% higher than the accuracy of the least squares model and 41.1% higher than the accuracy of the Bayes model.

When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.7$, the accuracy of the SCPDM is 54% higher than that of the least squares model and 60.7% higher than that of the Bayes model. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.4$, the accuracy of the SCPDM is 62.6% higher than the accuracy of the least squares model and 70.1% higher than the accuracy of the Bayes model. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.1$, the accuracy of the SCPDM is 42% higher than that of the least squares model and 43% higher than that of the Bayes model. When the parameters are set to $\sigma = 10$ and $|\mu_1 - \mu_2| = 1$, the accuracy of the SCPDM is 31% higher than the accuracy of the least squares model and 41.1% higher than the accuracy of the Bayes model.

When $\frac{|\mu_1 - \mu_2|}{\sigma} \le 1$, we consider $\Pr(\hat{t}^* = t^*)$. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 1$, the accuracy of the SCPDM is 25% higher than the accuracy of the least squares model and 33.6% higher than the accuracy of the Bayes model. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.7$, the accuracy of the SCPDM is 27.7% higher than that of the least squares model and 32.1% higher than that of the Bayes model. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.4$, the accuracy of the SCPDM is 23.2% higher than the accuracy of the least squares model and 28.8%

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higher than the accuracy of the Bayes model. When the parameters are set to $\sigma = 1$ and $|\mu_1 - \mu_2| = 0.1$, the accuracy of the SCPDM is 9.1% higher than that of the least squares model and 9.5% higher than that of the Bayes model. When the parameters are set to $\sigma = 10$ and $|\mu_1 - \mu_2| = 1$, the accuracy of the SCPDM is 9.2% higher than the accuracy of the least squares model and 9.5% higher than the accuracy of the Bayes model.

6. Conclusion

This paper focused on detecting the change points of WMDD. We conducted a certain asymptotic analysis and established an enhanced change point detection model (the SCPDM) based on the asymptotic results. According to theorem 2 (2), the enhanced sequence uses the significant variance information so that the weak mean difference increases from $|\mu_1 - \mu_2|$ to $2|\mu_1 - \mu_2|$, which makes the change point easier to detect and increases the accuracy of change point detection. In addition, for WMDD, the traditional methods may be improved by purely adding the sample capacity to the sequence. While under the premise of having the same amount of data, the SCPDM greatly increases the efficiency of change point detection by repeatedly detecting sequences with the same data structure. In addition, repeated measurements are possible for the life data of components at the same location. Hence, compared with traditional methods, the SCPDM can effectively detect change points. Although the accuracy of change point detection has been improved, this paper also has several limitations. First, we only discussed that $y_1, \ldots, y_T$ is independent with a normal distribution and a single change point. Second, the reason why the relationship between $|\mu_1 - \mu_2|$ and σ has an important influence on the accuracy of change point detection is not discussed in depth. We defined the ratio boundary of WMDD only by experience and simulations. In a future study, we will extend the SCPDM to other distribution types and multiple point detection. In addition, for **theorem 2**, we will reprove the theorem by introducing the relationship between $|\mu_1 - \mu_2|$ and σ.

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Figure Legends

Fig 1. Diagram of the accuracy of change point detection influenced by the variance. (A) $\sigma = 0.1$, $|\mu_1 - \mu_2| = 1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (C) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$.

Fig 2. Variation in the difference between adjacent parallel structural components.

Fig 3. The value of

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{1i}''(t^*,y'')\right\|-\left(\left|\mathring{\mu}_{1i}'(t^*,y')-\mathring{\mu}_{2i}'(t^*,y')\right|+\left|\mathring{\mu}_{1i}''(t^*,y'')-\mathring{\mu}_{2i}''(t^*,y'')\right|\right)\right)$$

Fig 4. The value of $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{1i}''(t^*,y'')\right\|\right)$ for 500 repetitions. (A) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.7$. (C) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (D) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$.

Fig 5. The value of $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t,y')-\dot{\mu}_{2i}'(t,y')\right|-\left|\dot{\mu}_{1i}''(t,y'')-\dot{\mu}_{1i}''(t,y'')\right\|\right)$ at location t. (A) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.7$. (C) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (D) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$. (E) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.4$.

Fig 6. Probability of each location becoming a change point in the least squares model. (A) $\sigma = 0.5$, $|\mu_1 - \mu_2| = 1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (C) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$. (D) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.4$. (E) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.7$. (F) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.1$.

Fig 7. Probability of each location becoming a change point in the Bayes model. (A) $\sigma = 0.5$, $|\mu_1 - \mu_2| = 1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (C) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$. (D) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.4$. (E) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.7$. (F) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.1$.

Fig 8. Probability of each location becoming a change point in the SCPDM. (A) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.1$. (B) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.7$. (C) $\sigma = 1$, $|\mu_1 - \mu_2| = 1$. (D) $\sigma = 10$, $|\mu_1 - \mu_2| = 1$. (E) $\sigma = 1$, $|\mu_1 - \mu_2| = 0.4$.

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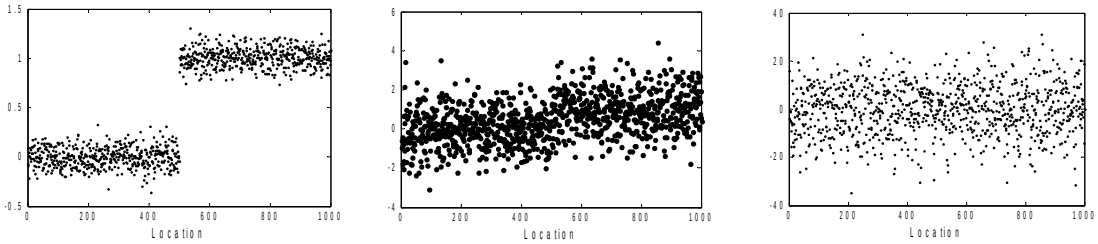
Figures



(a) $\sigma = 0.1, |\mu_1 - \mu_2| = 1$ (b) $\sigma = 1, |\mu_1 - \mu_2| = 1$ $\sigma = 10, |\mu_1 - \mu_2| = 1$

Fig.1 Diagram of the accuracy of change point detection influenced by variance.

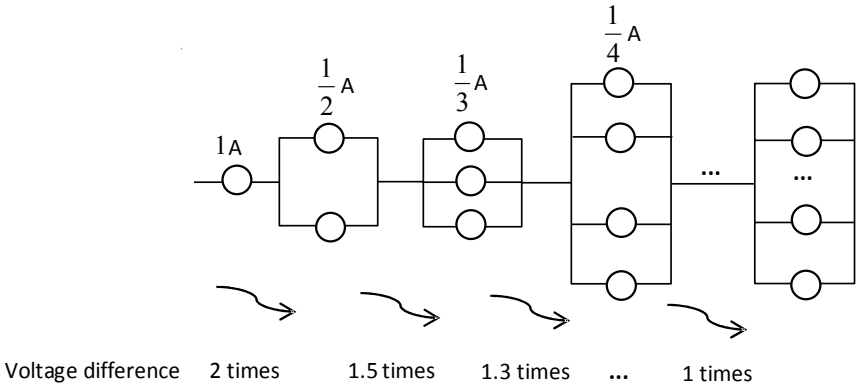
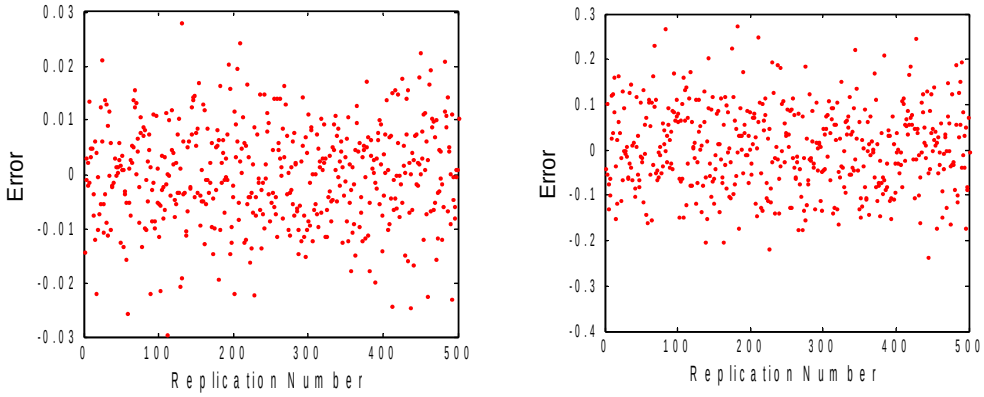


Fig. 2. Varieties in difference between adjacent parallel structural components



(a) $\sigma = 0.1, |\mu_1 - \mu_2| = 0.5$ (b) $\sigma = 1, |\mu_1 - \mu_2| = 5$

Fig. 3. Under 500 repetitions, value of

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left|\left|\dot{\mu}_{1i}'(t^*,y')-\dot{\mu}_{2i}'(t^*,y')\right|-\left|\dot{\mu}_{1i}''(t^*,y'')-\dot{\mu}_{1i}''(t^*,y'')\right|\right|-\left(\left|\mathring{\mu}_{1i}'(t^*,y')-\mathring{\mu}_{2i}'(t^*,y')\right|+\left|\mathring{\mu}_{1i}''(t^*,y'')-\mathring{\mu}_{2i}''(t^*,y'')\right|\right)\right)$$

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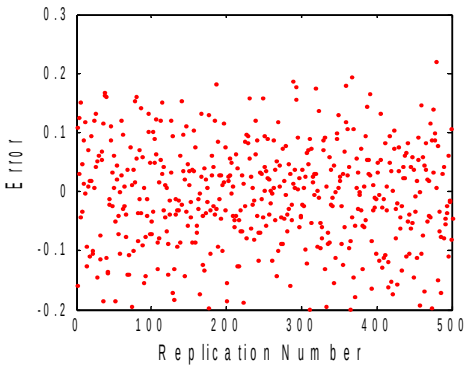
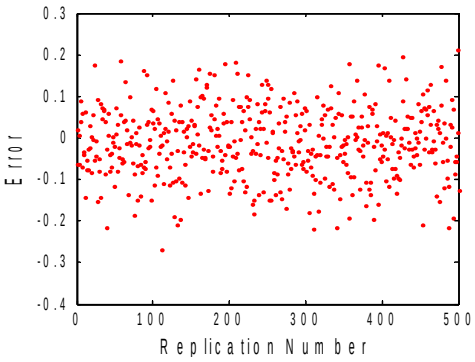
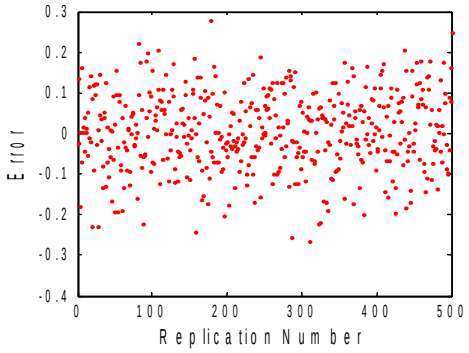


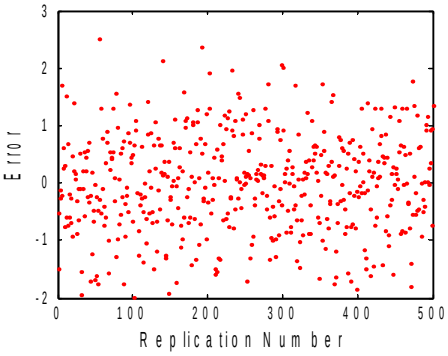
(a) $\sigma = 1, |\mu_1 - \mu_2| = 0.1$



(b) $\sigma = 1, |\mu_1 - \mu_2| = 0.7$



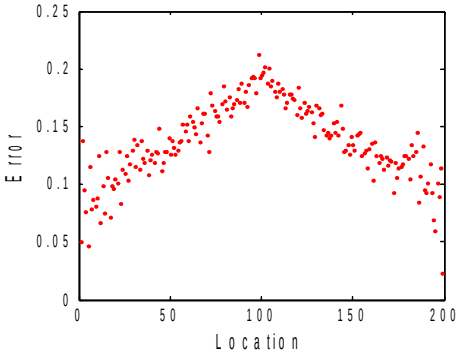
(c) $\sigma = 1, |\mu_1 - \mu_2| = 1$



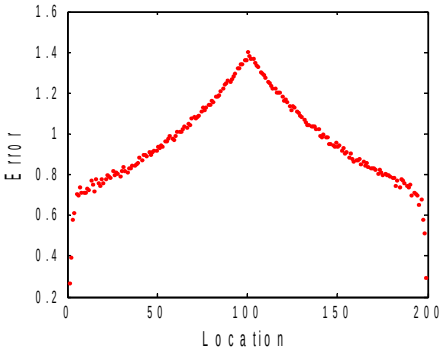
(d) $\sigma = 10, |\mu_1 - \mu_2| = 1$

Fig. 4. Under 500 repetitions, value of

$$\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\left|\dot{\mu}_{1i}{}'(t^*, y') - \dot{\mu}_{2i}{}'(t^*, y')\right| - \left|\dot{\mu}_{1i}{}''(t^*, y'') - \dot{\mu}_{1i}{}''(t^*, y'')\right|\right\|\right)$$



(a) $\sigma = 1, |\mu_1 - \mu_2| = 0.1$



(b) $\sigma = 1, |\mu_1 - \mu_2| = 0.7$

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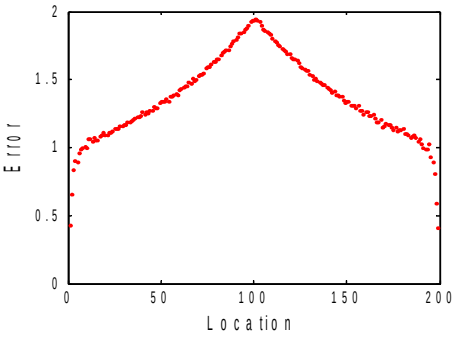
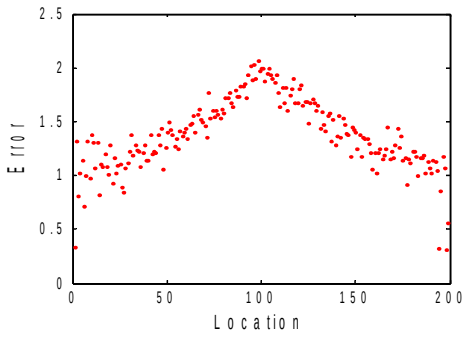
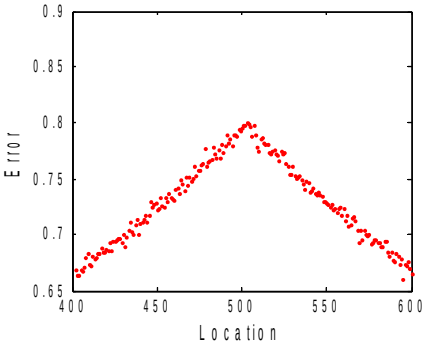


(c) $\sigma = 1, |\mu_1 - \mu_2| = 1$

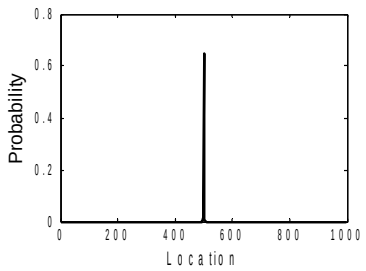


(d) $\sigma = 10, |\mu_1 - \mu_2| = 1$

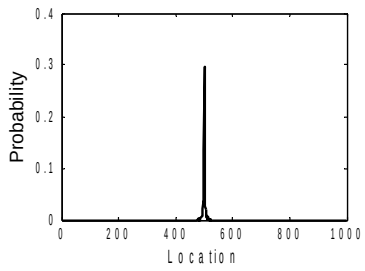


(e) $\sigma = 1, |\mu_1 - \mu_2| = 0.4$

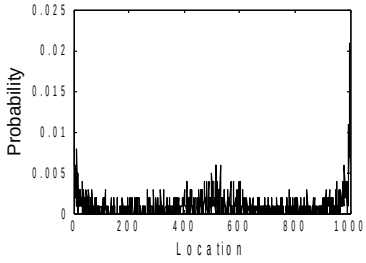
Fig. 5. The value of $\frac{1}{n_c}\sum_{i=1}^{n_c}\left(\left\|\dot{\mu}_{1i}'(t,y') - \dot{\mu}_{2i}'(t,y')\right| - \left|\dot{\mu}_{1i}''(t,y'') - \dot{\mu}_{1i}''(t,y'')\right\|\right)$ at location.



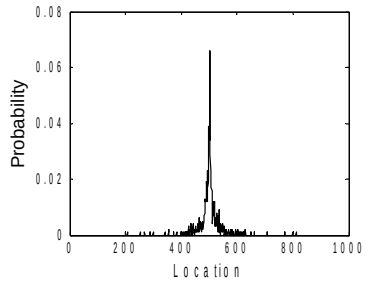
(a) $\sigma = 0.5, |\mu_1 - \mu_2| = 1$



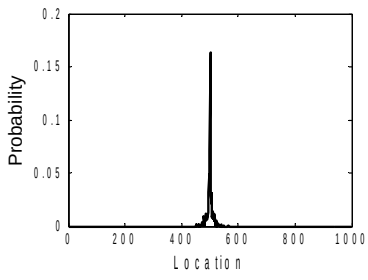
(b) $\sigma = 1, |\mu_1 - \mu_2| = 1$



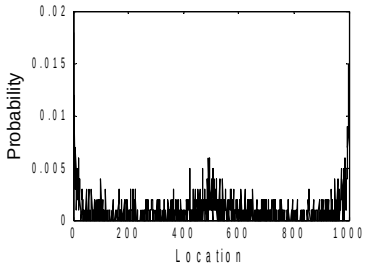
(c) $\sigma = 10, |\mu_1 - \mu_2| = 1$



(d) $\sigma = 1, |\mu_1 - \mu_2| = 0.4$



(e) $\sigma = 1, |\mu_1 - \mu_2| = 0.7$



(f) $\sigma = 1, |\mu_1 - \mu_2| = 0.1$

Fig.6. Probability of the each location becoming a change in Least Square Model

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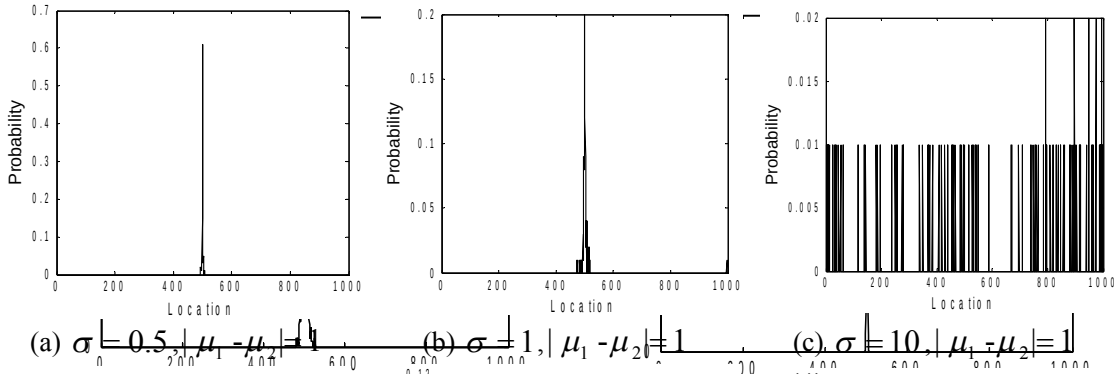
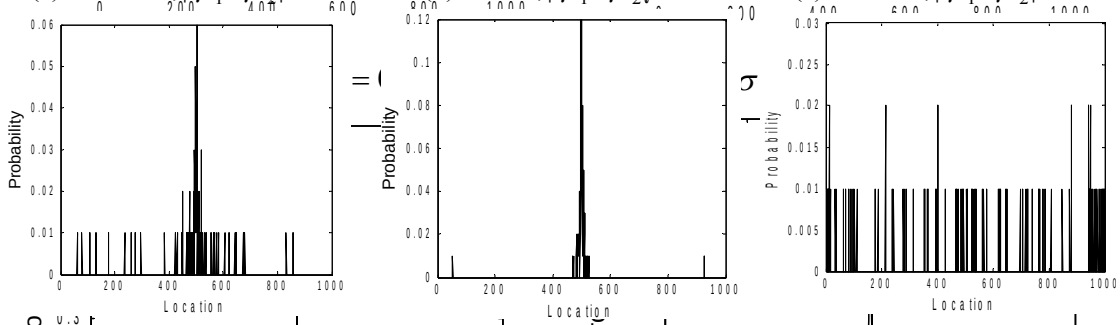
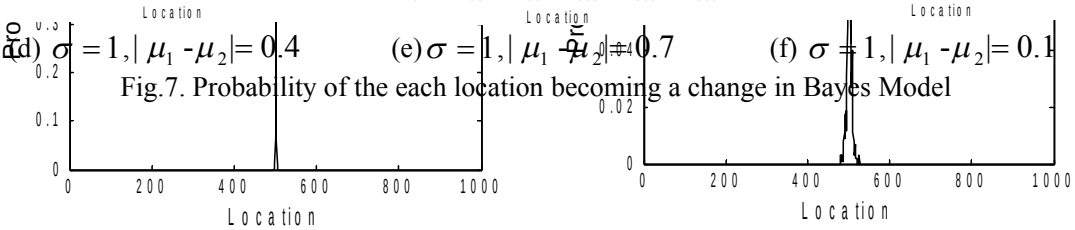


(a) $\sigma = 0.5, |\mu_1 - \mu_2| = 1$ (b) $\sigma = 1, |\mu_1 - \mu_2| = 1$ (c) $\sigma = 10, |\mu_1 - \mu_2| = 1$

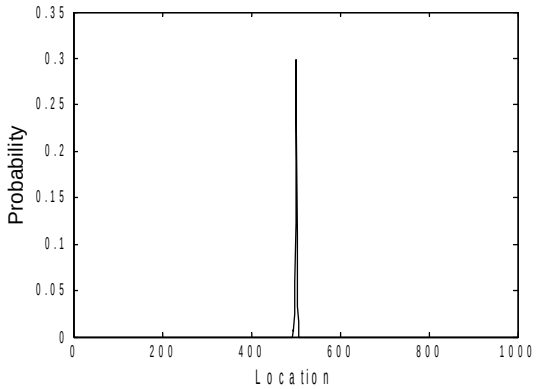


(d) $\sigma = 1, |\mu_1 - \mu_2| = 0.4$ (e) $\sigma = 1, |\mu_1 - \mu_2| = 0.7$ (f) $\sigma = 1, |\mu_1 - \mu_2| = 0.1$

Fig.7. Probability of the each location becoming a change in Bayes Model



(c) $\sigma = 1, |\mu_1 - \mu_2| = 1$ (d) $\sigma = 10, |\mu_1 - \mu_2| = 1$



(e) $\sigma = 1, |\mu_1 - \mu_2| = 0.4$

Fig. Probability of the each location becoming a change in SCPDM

格式要求：

图字、表字；图题、表题；图注和表注：TNR， 8 pt。

图表尺寸：单栏最大：185 mm × 120 mm；双栏最大：宽不超过 60 mm。

图清晰度：300 dpi 或以上。若图中有多个图表，用 A、B、C 等分别表示。

注明图（横纵坐标）表的度量衡单位及其说明。

按照正文中引用的顺序升序排列；每个图单独占据一页，不同图之间用分页符。

公式：

公式放在正文中，需用公式编辑器进行编辑。

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Tables

Table.1 Estimate probability of a change in certain intervals of three models.

Model	$\lvert \mu_1 - \mu_2 \rvert$	σ	Change point location						
			484-488	489-493	494-498	499-503	504-508	509-513	514-518
Least Squared Method	1	0.5	0	0.001	0.028	0.942	0.029	0	0
	1	1	0.010	0.031	0.113	0.681	0.113	0.021	0.013
	1	10	0.006	0.008	0.010	0.019	0.013	0.007	0.012
	0.4	1	0.054	0.075	0.113	0.215	0.107	0.055	0.043
	0.7	1	0.039	0.048	0.166	0.417	0.148	0.048	0.034
	0.1	1	0.009	0.014	0.015	0.010	0.015	0.011	0.009
Bayes Method	1	0.5	0	0	0.07	0.87	0.06	0	0
	1	10	0.01	0.01	0.01	0.01	0	0	0.01
	1	1	0.01	0.02	0.11	0.58	0.13	0.07	0.03
	0.7	1	0.06	0.06	0.18	0.35	0.18	0.07	0.01
	0.1	1	0.01	0.03	0.01	0	0.02	0	0
	0.4	1	0.02	0.04	0.08	0.14	0.10	0.06	0.02
SCPDM Method	1	1	0	0	0.001	0.991	0.008	0	0
	1	10	0.019	0.065	0.171	0.430	0.201	0.073	0.023
	0.7	1	0	0	0.010	0.957	0.033	0	0
	0.1	1	0.019	0.065	0.171	0.430	0.201	0.073	0.023
	0.4	1	0	0	0.040	0.841	0.118	0.001	0

Table. 2. Estimate probability of $\hat{t}^* = t^*$ of three models

MD	SD	Least Squared Method	Bayes Method	SCPAM
1	0.5	0.651	0.61	---
1	1	0.286	0.2	0.536
1	10	0.003	0	0.095
0.4	1	0.066	0.01	0.298
0.7	1	0.164	0.12	0.441
0.1	1	0.004	0	0.095

要求：表头、表题和表均放置在此处。不同表占据不同页码，表之间用分页符分开。

采用三线表。要求简洁、自明性强。

图字、表字；图题、表题；图注和表注：TNR， 8 pt。

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图表尺寸：单栏最大：185 mm × 120 mm；双栏最大：宽不超过 60 mm。

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